Time Value of Money Concepts (Present/Future Value)

The time value of money refers to interest rates and the effect they have on the value of money over time. Interest is the fee paid for having the use of money. Interest is earned by investors on money that has been invested in interest-earning securities (usually bonds) and by savers in bank accounts that earn interest. Borrowers pay interest to lenders. For businesses, the lenders are the holders of their debt securities and/or their banks. Individuals pay interest on school loans, car loans, mortgages, and credit card debt (purchases not paid off within the credit card's grace period).

When money is borrowed or lent/invested, the amount of the loan or investment is called the **principal**. **Interest** is calculated as a proportion of the principal for the period during which the money is used. The **interest rate** is the rate at which the interest accumulates. It is usually stated as a percentage of the principal per period. Usually, interest rates are quoted as annual interest rates (for example, 6% per annum) but they may also, particularly for consumer debt, be quoted per month (such as 1% per month). An interest rate of 1% per month is equivalent to 12% per annum.

Simple Interest

Simple interest is interest that is incurred only on the amount of principal that is outstanding. Unpaid interest is not added to the principal, and interest is not charged on the unpaid interest.

Simple interest for any amount of time can be calculated with this formula:

$$I = P \times IR \div DY \times DO$$

Where:	**I**	=	Simple interest incurred
	P	=	Principal outstanding
	IR	=	Interest rate per year (per annum), in decimal form
	DY	=	Number of days in year (365 or 3601)
	DO	=	Number of days the principal is outstanding

P × IR ÷ DY equals the amount of interest charged per day. Multiplying the daily interest by the number of days the principal is outstanding equals the interest incurred for the period the principal is outstanding.

Note: Time value of money tables are available to download from:
https://www.hockinternational.com/download/PVFV_Tables.xlsx

1 The default number of days per year used on the CMA exams is 365. 365 days should be used unless an exam question specifies 360 days.

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Example: The principal of a loan is $100,000. The annual simple interest rate is 6%. Therefore, if the entire $100,000 were to remain outstanding (not repaid) for one full year, the amount of **simple interest** owed by the borrower for the year would be $100,000 × 0.06, or $6,000.

For each **day** the $100,000 remains outstanding, the amount of simple interest owed by the borrower is $100,000 × 0.06 ÷ 365, or $16.44 (rounded).

Or, if interest is to be calculated monthly, the annual amount of interest may be divided by 12 to calculate the amount of interest charged monthly.

If the $100,000 loan is outstanding for 15 days and is then repaid, the total simple interest owed by the borrower will be $100,000 × 0.06 ÷ 365 × 15, or $246.58. The amount the borrower would repay would be $100,246.58.

If the loan is outstanding for one full year, the interest charged will be $100,000 × 0.06 ÷ 365 × 365, or simply $100,000 × 0.06, which is $6,000. The borrower would repay $106,000.

Simple interest for a period that is greater than one year can be calculated using the above formula, or it can be calculated more simply. Using the above formula, a $100,000 loan for 2 years (730 days) at 6% simple interest would incur interest of $100,000 × 0.06 ÷ 365 × 730, which equals $12,000. However, a simpler way to calculate the same amount would be $100,000 × 0.06 × 2 = $12,000. Once one year's interest has been calculated (here, $100,000 × 0.06), it can be adjusted to a period of less than one year or to a period of greater than one year, assuming the principal balance does not change during the period the loan is outstanding.

Compound Interest

Usually, interest is **compounded** at regular intervals. When interest is compounded, interest that has accrued (been incurred) and has not yet been paid by the borrower is added to the outstanding principal at the end of each stated compounding period. Then the interest incurred for the next period is calculated based on the increased principal balance that consists of the previous principal plus the compounded (added) interest. The amount of interest calculated using this procedure is called **compound interest**, and it is higher than simple interest.

Example: $100,000 has been deposited in a bank that pays 2% interest per annum, compounded quarterly. At the end of the first quarter containing 90 days, interest will be calculated as follows:

$$\mathbf{I = P \times IR \div DY \times DO}$$

$$I = \$100{,}000 \times 0.02 \div 365 \times 90, \text{ or } I = \$493.15$$

Alternatively, interest for one quarter can be approximated this way:

$$I = \$100{,}000 \times 0.02 \div 4, \text{ or } I = \$500$$

The difference, of course, is in the way the interest is calculated. In the first way, it is calculated as a daily interest amount for each quarter. In the second way, it is calculated for one-fourth of a year for each quarter. But over the course of one full year, the annual amount of interest will be the same.

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Example (continued):

Assuming the interest is calculated the first way, the $493.15 interest accrued at the end of the first quarter will be added to the $100,000 to create a new principal balance of $100,493.15 at the beginning of the second quarter. Interest for the second quarter the funds are on deposit, containing 91 days, will be based on this new, increased principal amount:

$$I = \$100{,}493.15 \times 0.02 \div 365 \times 91 = \$501.09.$$

After the second quarter's interest has been added to the principal, the principal on deposit during the third quarter will be $100,493.15 + $501.09 = $100,994.24.

Interest for the third quarter containing 92 days will be

$$\$100{,}994.24 \times 0.02 \div 365 \times 92 = \$509.12$$

The new principal balance after compounding of the interest will be $100,994.24 + $509.12 = $101,503.36.

Interest for the fourth quarter containing 92 days will be

$$\$101{,}503.36 \times 0.02 \div 365 \times 92 = \$511.69$$

The new principal after compounding of the interest will be $101,503.36 + $511.69 = $102,015.05.

If the depositor withdraws the full balance in the account after one year and closes the account, the deposit will have earned a total of $2,015.05 on the original $100,000 deposit. The Annual Percentage Rate (APR), or effective annual interest rate, earned is 2.015% ($2,015.05 ÷ $100,000.00). The APR is higher than the simple interest rate of 2% because of the compounding. Compound interest means interest on interest.

If the interest had been compounded monthly instead of quarterly, the total interest earned and the effective annual interest rate would have been even greater.

Note: For a stated interest rate and a stated period, compound interest is greater than simple interest, because interest is earned on interest; and the more frequently interest is compounded, all other things being equal, the greater will be the amount of interest earned.

Present Value

Present value computations are used to look for an **unknown present value** of a known single amount of money or stream of equal payments that will be either paid or received in the future when the interest rate that could be received by investing the receipts or payments (if they were owned or paid now instead of in the future) is known. Calculating the present value of a future amount is called **discounting** the future amount from the future to the present. The present value of an amount or amounts to be received or paid in the future is less than the future amount or amounts because of the forgone interest. The present value of an amount to be received in the future is the amount that would grow to the future, known, amount, if it were owned today and could be invested at the specific known interest rate.

The present value of future cash flows is an underlying concept used in capital budgeting, valuing debt securities, and many other finance and accounting processes. It is critical to understand how to work with and calculate the present value of future cash flows. On the exam, when there is a question that requires a present value calculation, present value tables are made available.

Tables for finding the present value of a single sum to be paid or received in the future, or for a series of equal payments to be paid or received in the future, are available for reference at the link on the first page of this document.

The factors in the present value tables are for the present values of amounts of 1, so the appropriate factor for the given term and interest rate is multiplied by the amount to be paid or received in the future to find the present value of that future amount.

The present and future value tables are usually used under the assumption that the periods are one year in length, that is, the interest rates are for one year and the interest is compounded once a year, at the end of the year. However, the tables can be used for more frequent compounding and (for annuities) for more frequent payments. The interest rate used simply needs to be adjusted to its equivalent for the period being used. For example, an annual interest rate of 12% is equivalent to an interest rate of 1% per month, and the number of periods used needs to be the number of months instead of the number of years.

If the factor needed is not available in a table, the factor can be calculated. The formulas needed to calculate the factors are given in this document. Present values and future values can also be calculated on a financial calculator. Four models of financial calculators are permitted on the CMA exam but are not required.

Note: The four models of financial calculators permitted on the CMA exams are:

- Texas Instruments BA II Plus (**not** the BA II Plus Professional),
- HP 12c, and
- HP 12c Platinum.
- The HP 10BII is also valid for use on the exam but is no longer available for purchase.

Present Value of 1 (A Single Amount)

The present value (PV) of a single amount to be received in the future is the current value of the future monetary receipt, recognizing that if one must wait to receive that amount, the opportunity to earn interest is lost because the money is not available to deposit or invest now. Waiting to receive the money causes a loss in its value because of the opportunity loss, so the current worth of the future monetary receipt will be lower than the future receipt amount. How much lower it will be will depend on the interest rate at which the forgone interest could have been earned. The higher the forgone interest rate, the lower the current value, or present value, of the future receipt is. The interest rate used is called the **discount rate**, and the present value of the future amount is also called a **discounted** amount.

Exam Tip: The present value of a future cash flow decreases as the interest rate used to discount it increases. The present value of a future cash flow also decreases as the time to wait before receiving it increases.

Example: Jane Brown will receive $10,000 in one year's time, but if she had the money today, she could earn 3% annual interest on it. The present value of that $10,000 is the amount she could invest today at 3% if she had it, and with the interest added, it would grow to $10,000 in one year's time.

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If she had the money and could invest it today at 3%, the amount she would need to invest today to have $10,000 in one year's time would be greater than the amount she would need to invest today to have $10,000 in one year's time if she could earn 4% on the money instead. For that reason, **the present value of a future cash flow decreases as the discount rate increases**. The present value of a future cash flow discounted at 4% is lower than the present value of the same amount discounted for the same term at 3%.

The amount of time to wait before receiving a future cash flow also affects the present value of the future cash flow.

If Jane Brown will receive the $10,000 in five years, the amount that she would need to invest today at 3% to have $10,000 in five years will be less than the amount she would need to invest today at 3% to have $10,000 in one year, because the opportunity loss is five years of interest not earned instead of only one year. Thus, **the present value of a future cash flow decreases as the time to the receipt of the money increases**.

Note: The present value of a future amount depends on both the **discount rate** and the **number of periods** from the present date to the future date when the money will be received.

To calculate the present value of a single amount, use the Present Value of 1 table available at the link on the first page of this document. Look across the top of the columns to find the interest rate and then look down that column to find the number of periods. The factor at the intersection of the rate and the time represents the present value of 1 at that rate and for that period. To calculate the present value of an amount of X, multiply that X amount by the appropriate PV of 1 factor obtained from the table.

$$\textbf{Present Value} = \textbf{Future Amount} \times \textbf{Present Value of 1 Factor for } n \textbf{ periods at } i \textbf{ interest}$$

Derivation of the Factor for the Present Value of 1

The present value of a single future amount can be calculated without resorting to a factor table. If the discount rate or the term needed does not appear in a factor table, the factor can be calculated as follows:

$$\text{Present Value of 1} = \frac{1}{(1+i)^n}$$

The formula will result in a factor that is carried out to several decimal places. The calculated factor will be more accurate than the factor in a factor table because the factors in factor tables are usually rounded to three decimal places.

Example: The present value of $100,000 discounted for one year at 9% is:

$$\$100{,}000 \times \frac{1}{1.09} = \$91{,}743$$

The present value of $100,000 discounted for two years at 9% is:

$$\$100{,}000 \times \frac{1}{1.09^2} = \$84{,}168$$

The rounded factors appearing in the PV of 1 table are 0.917 for one year and 0.842 for two years, so when the factors are used to find the present values of $100,000 discounted for one year and two years, the resulting present values are $91,700 and $84,200, respectively. However, the differences between those present values and the present values calculated using the above formula are not material.

Note: All the time value of money factors (Present Value of 1, Present Value of an Annuity, Future Value of 1, Future Value of an Annuity) represent the present or future value of 1 or an annuity of 1. Therefore, to use a factor to find the present value of an amount greater than 1, such as 100,000, multiply the amount by the appropriate factor.

Present Value of an Annuity (a Stream of Payments)

An annuity is a constant stream of payments of the same amount either paid or received regularly over several periods at the same point in each period, such as at the end of each year.

The **present value** of an annuity is the current value of a stream of equal payments that will be paid or received in the future over a designated period, with the understanding that money paid or received in the future is worth less than money paid or received today because of the effect of the forgone interest. The present value of an annuity can be found using the Present Value of an Annuity factor table available at the link on the first page of this document. The present value of an annuity factor can be used to calculate the present value of an annuity **if and only if** all the following are true:

1) The amount to be received or paid is a **constant** (**that is**, **the same**) **amount** for every payment.
2) The amount will be received or paid **at the same point in every period**.
3) The interest will be compounded **once each period**.

Ordinary Annuities and Annuities Due

There are two types of annuities, ordinary annuities, and annuities due:

1) An **ordinary annuity** (also called an annuity in arrears) is an annuity with payments made or received at the **end** of each period. The factors in a Present Value of an Annuity table are for ordinary annuities, as is the table available at the link on the first page of this document.

Present value of an ordinary annuity	**=**	**Periodic payment × PV of an Ordinary Annuity Factor for *n* periods at *i* interest**

2) An **annuity due** is an annuity with payments made or received at the **beginning** of each period. To calculate the present value of an annuity due, use the ordinary annuity factor for the interest rate and **one period less** and **add 1.000 to it.**

Present value of an annuity due	**=**	**Periodic payment × PV of an Ordinary Annuity Factor for *n−1* periods at *i* interest + 1.000**

Example #1: Find the present value of an **ordinary** annuity of $1,000, with payments made or received at the **end** of each period, for four years at a 16% discount rate.

The factor for the present value of an ordinary annuity for four years at 16% is 2.798. The present value of a $1,000 ordinary annuity for four years with annual payments made or received at the **end** of each year at an annual interest rate of 16% is:

$1,000 × 2.798 = **$2,798**

Example #2: Find the present value of an **annuity due** of $1,000 for four years at a 16% discount rate, where the payments are made at the **beginning** of each period.

The present value of a $1,000 annuity due, with payments made at the **beginning** of each of four years, discounted at 16%, is calculated using the present value of an **ordinary** annuity factor for **three** years—**one period less**—at 16%, which is 2.246, and adding 1 to the factor.

Present value of an annuity due = $1,000 × (2.246 + 1.000) = **$3,246**

When the rate and the term are equal, the present value of an annuity due received at the beginning of each period is higher than the present value for the same amount received as an ordinary annuity at the end of each period.

Calculating a Loan Payment Amount

The present value of an ordinary annuity factor table such as the one available at the link on the first page of this document can be used to calculate loan payments when the payments are to be made once a year at the end of the year. The annual loan payment is the annuity. The principal balance of the loan is the present value of the annuity.

Example: The annual loan payment on a $100,000 five-year loan at an interest rate of 6% per annum with payments due at the **end** of each of five years (an ordinary annuity) is calculated as follows:

1) The PV of an Ordinary Annuity factor for 6% for five years is 4.212.

2) Divide the loan principal, $100,000, by the factor, 4.212.

$$\frac{\$100{,}000}{4.212} = \mathbf{\$23{,}741.69}$$

The result, $23,741.69, is the annual loan payment that will fully amortize the loan principal over five years with interest at 6%. The $23,741.69 is the annuity amount.

Finding a Loan's Beginning Principal Amount

If the amount of the annual payment, the term of the loan, and the interest rate are known, the loan's beginning principal can be calculated. The principal amount will be the present value of the stream of annuity payments.

Example: The annual payment amount is $23,741.69, due at the end of each year for five years. The interest rate on the loan is 6% per annum. What is the principal amount of the loan?

$23,741.69 × 4.212 = **$100,000.00** (rounded)

How much interest will be paid over the life of the loan?

The total interest paid is the difference between the total payments paid (including principal and interest) over the life of the loan and its original principal.

($23,741.69 × 5) − $100,000.00 = **$18,708.45**

The borrower will pay $18,708.45 in interest over the term of the loan.

Finding the Interest Rate on a Loan

If the amount of the annual payment, the principal amount, and the term are known, the interest rate on the loan can be calculated.

Example: The annual loan payment on a $25,000 loan is $5,935, due at the **end** of each year for a term of five years. What is the interest rate on the loan?

This is an ordinary annuity because the payments are made at the **end** of each year. The present value of an ordinary annuity is the annuity amount multiplied by the appropriate Present Value of an Ordinary Annuity factor. The present value of the annuity is $25,000 and the annuity amount is $5,935. Once the factor is known, it can be located in a Present Value of an Ordinary Annuity table and used to find the interest rate.

1) To find the factor for the loan, divide $25,000 by the loan payment, $5,935.

   $25,000 ÷ $5,935 = 4.212.

2) On a PV of an Ordinary Annuity table, available at the link on the first page of this document, on the five-year line find either 4.212 or the factor that is closest to 4.212. The factor in the 6% column is exactly 4.212. Therefore, the interest rate on this loan is exactly **6%**.

If the factor for the loan falls in between two factors on the factor table, use interpolation to estimate the interest rate. The rate will be in between the two rate columns in the table.

Finding a Single Amount to Deposit to Provide for a Series of Future Withdrawals

The present value of an annuity can also be used to find a single amount that, if invested at a given compound interest rate now, will provide for a series of a certain number of future withdrawals of a certain amount.

Example: Samson wants to withdraw $20,000 per year for five years at the end of each year from an account. The account pays 5% interest during that period, compounded annually. How much does he need to invest today to be able to withdraw $20,000 per year and empty the account by the end of the five years?

The PV of an Ordinary Annuity factor for 5% for five years is 4.329. The present value of a stream of payments of $20,000 at the end of each year for five years is

$$\$20{,}000 \times 4.329 = \mathbf{\$86{,}580}$$

Therefore, Samson needs to invest $86,580 today to be able to withdraw $20,000 per year for five years beginning in one year.

Derivation of the Factor for the Present Value of an Ordinary Annuity

The factor for the present value of an ordinary annuity can be calculated with the following formula:

$$\text{Present Value of an Ordinary Annuity} = \frac{(1 + i)^n - 1}{i\,(1 + i)^n}$$

Example: The present value factor for a five-year ordinary annuity discounted at 5% is calculated as follows:

$$\text{Present Value of an Ordinary Annuity at 5\% for 5 years} = \frac{(1 + 0.05)^5 - 1}{0.05\,(1 + 0.05)^5}$$

$$= \frac{1.27628 - 1}{0.05 \times 1.27628} = \mathbf{4.3294575}$$

The factor for five years at 5% from the PV of an Ordinary Annuity factor table is 4.329.

Payments Made or Received More Frequently Than Once a Year

In the present value factor tables, available at the link on the first page of this document, payments are assumed to be made or received once a year. However, the factors in the Present Value of 1 or the Present Value of an Ordinary Annuity table can be adjusted for use when payments are made more frequently than once a year (such as quarterly or monthly) by adjusting the rate and the number of periods used, as follows:

- The annual rate is divided by the number of payments per year. For example, a 12% annual rate compounded monthly becomes 1% per month.
- The number of periods is the number of years multiplied by the number of payments per year.
  - The present value of a single amount to be received in two years discounted at an 8% annual rate with interest compounded quarterly is the amount to be received discounted at 2% (8% ÷ 4) for 8 periods (2 × 4).
  - The present value of a payment to be received twice a year for five years at a discount rate of 6% annually is the amount of one payment multiplied by the present value of an annuity factor for 3% (6% ÷ 2) for 10 (5 × 2) periods.

Example: The annual interest rate is 4% and the interest is received and reinvested (compounded) quarterly. One year contains four quarterly periods, so the rate of return per quarter is 4% ÷ 4, or 1% per quarter.

To find the present value of $10,000 to be received in one year's time when the interest rate is 4% compounded quarterly, on the Present Value of 1 table look for the discount rate of **1%** and then look down the column under 1% to the line for **four periods**. The factor for 1% for four periods is 0.961. Therefore, the present value of $10,000 discounted at 4% for one year with interest compounded quarterly is $10,000 × 0.961, or **$9,610**.

Contrast that with the present value of $10,000 invested at 4% but with interest compounded annually. The present value factor for 4% for one period is 0.962, so the present value of $10,000 discounted at 4% for one year with interest compounded annually is $10,000 × 0.962, or **$9,620**. An investment of $9,620 invested at 4% interest with interest compounded annually would also grow to $10,000 in one year.

The required investment (the present value) is less when the interest is compounded quarterly than when the interest is compounded annually, yet both investments will be worth $10,000 after one year's time. That occurs because more interest will be earned on the investment when the interest is compounded quarterly than when it is compounded annually.

Present Value of a Series of Payments of Varying Amounts

If annual amounts are to be received and the amounts vary each year, the present value of the varying cash flows is the sum of the present values of the individual cash flows, calculated using the Present Value of 1 factors for the interest rate and the number of years until the receipt of each individual cash flow.

Example: Annual amounts to be received are $8,000 the first year, $9,000 the second year, $10,000 the third year, $11,000 the fourth year, and $12,000 the final year. The discount rate is 5%. The present value of the varying cash flows is the sum of the present values of each of the individual cash flows, as follows:

Year 1:	$8,000 × 0.952	$ 7,616
Year 2:	$9,000 × 0.907	8,163
Year 3:	$10,000 × 0.864	8,640
Year 4:	$11,000 × 0.823	9,053
Year 5:	$12,000 × 0.784	9,408
	Present value	**$ 42,880**

Future Value

Future value computations are used to look for an **unknown future value** of a known single amount of money or stream of equal payments paid or received that earn a given interest rate over a specific period. When solving for future value, a single cash flow or a series of equal cash flows are accumulated, with interest, to a future point. The future value will be greater than the single cash flow or the total of the series of cash flows because of the interest added to the principal each compounding period.

The concept of future value is less likely to be asked on an exam than present value, but it is possible that a question will require its use.

Future Value of 1 (a Single Amount)

The future value of a single amount is used to answer a question such as "If an amount such as $100,000 is invested now at an annual interest rate of 6% compounded annually, how much will there be at the end of five years"? The future value of a single amount can also be used to find the amount that needs to be invested to have a specified amount at the end of a year at a specified annual interest rate.

To calculate the future value of any single amount, use the Future Value of 1 table available at the link on the first page of this document. Look across the column headings to find the appropriate interest rate and then look down that column to find the number of periods. The factor at the intersection represents the future value of 1 invested at that interest rate for that term. To calculate the future value of an amount of X, multiply that X amount by the future value factor obtained from the table.

Future Value = Present Value × Future Value of 1 Factor for *n* periods at *i* interest

Example:

If John invests $100,000 now for five years and does not touch it, and he receives interest on it compounded annually at a rate of 6% per annum, how much will he have at the end of five years?

On a Future Value of 1 table, the factor in the 6% column on the 5-year line is 1.338. The future value of $100,000 invested for five years at an interest rate of 6% per annum, compounded annually, is

Future value = $100,000 × 1.338 = **$133,800**

To find how much needs to be invested (the present value) to have a specified amount at the end of a specified period, restate the formula as follows:

Present Value = Future Value ÷ Future Value of 1 Factor for *n* periods at *i* interest

Example:

John decides he wants to have $150,000 at the end of five years, and he wants to know how much he needs to invest now at 6% compounded annually to have that amount at the end of five years.

On a Future Value of 1 table, the factor in the 6% column on the 5-year line is 1.338. The present value that he needs to invest now for five years at an interest rate of 6% per annum, compounded annually, to have a future value of $150,000 is

Present value = $150,000 ÷ 1.338 = **$112,108**

Note: The Future Value of 1 table provided at the link on the first page of this document assumes that payment is made or received at the **beginning** of the first period.

Derivation of the Factor for the Future Value of 1

If the needed interest rate or term does not appear in a factor table, the formula to use to calculate the factor for the Future Value of 1 is

$$\text{Future Value of 1} = (1 + i)^n$$

Example: Determine the factor to use to calculate the future value of $100,000 invested for five years at an interest rate of 6% per annum, compounded annually.

Future Value of 1 at 6% for 5 years = $(1 + 0.06)^5$ = **1.338225577**

The factor from the Future Value of 1 factor table for 6% for five years is 1.338.

Future Value of an Annuity (a Stream of Payments)

The Future Value of an Annuity is used to answer the question "If an amount such as $20,000 is invested annually for five years at an annual interest rate of 6% compounded annually, how much will there be at the end of five years"?

A future value calculation can also be used to determine the amount that needs to be set aside from current earnings **each year** for a specified number of years to have a given needed amount at the end of the period, assuming a certain interest rate to be received on the deposited funds.

The future value of an annuity is the accumulated sum of all the equal payments made plus the accumulated compound interest on the payments during the accumulation period.

The Future Value of an Annuity factor available at the link on the first page of this document can be used to calculate the future value of an annuity **if and only if** all the following are true:

1) The amount to be received or paid is a **constant** (**that is**, **the same**) **amount** for every payment.
2) The amount will be received or paid **at the same point in every period**.
3) The interest will be compounded **once each period**.

Ordinary Annuity Versus Annuity Due

The payments for a future value of an annuity may be made either at the beginning of each period or at the end of each period.

- When the payments are made at the **end** of each period, the annuity is called an **ordinary annuity.**
- When the payments are made at the **beginning** of each period, the annuity is called an **annuity due**.

Usually, a Future Value of an Annuity factor table is for an ordinary annuity, meaning that it assumes that the first annuity payment will not be made until the **end** of Year 1. However, a Future Value of an Annuity factor table **may be** for an annuity due instead, meaning that it assumes the first payment is made at the **beginning** of Year 1.

To determine whether a Future Value of an Annuity factor table is for an ordinary annuity or an annuity due, look at the factors for Period 1 all the way across the table.

1) If the factors for Period 1 for all interest rates are 1.000 all the way across the first line of the factor table, the table is a Future Value of an **Ordinary Annuity** factor table. When an annuity is an ordinary annuity and the payments are made at the **end** of each period, no interest will be earned during Period 1 because no payment will be made until the end of Period 1. Therefore, when the first annuity payment is made at the end of Period 1, the balance in the account at the end of Period 1 will be the same as the first amount deposited, with no interest earned or compounded. Thus, the factor for Period 1 is 1.000.

2) If the factors for Period 1 for all interest rates are 1 + the interest rate, the table is a Future Value of an **Annuity Due** factor table. When an annuity is an annuity due and the payments are made at the **beginning** of each period, the first payment will be made at the beginning of Period 1. Therefore, at the end of Period 1, interest for one period will have been earned and added to the principal that was deposited at the beginning of the period (that is, compounded). A Future Value of an **Annuity Due** factor table may also be called a "Compound Sum of an Annuity" factor table.

Note: The Future Value of an Annuity Table available at the link on the first page of this document is for an **ordinary annuity**, which assumes that payments are made at the **end** of each period.

Calculating the Future Value of an Ordinary Annuity

To calculate the future value of an ordinary annuity (payments made at the **end** of each period), **if a Future Value of an Ordinary Annuity factor table is available**, use the ordinary annuity factor from the table as follows:

Future value of an ordinary annuity	**=**	**Periodic payment × FV of an Ordinary Annuity Factor for *n* periods at *i* interest**

If the only table available is a Future Value of an Annuity Due (which assumes payments are made at the **beginning** of each period), to calculate the future value of an ordinary annuity, use the annuity due factor from the table for **one period less** and **add 1.000** to it.

Future value of an ordinary annuity	**=**	**Periodic payment × FV of an Annuity Due Factor for *n*−*1* periods at *i* interest + 1.000**

Example: To find the future value of an ordinary annuity of $10,000 with payments made at the **end** of each period for three years at an interest rate of 5% when the Future Value of an Annuity table available is for an annuity due (for payments made at the **beginning** of the periods):

1) Find the annuity due factor for **two** years at 5% in the annuity due table: The annuity due factor for two years at 5% is 2.153.

2) Add 1.000 to the factor: 2.153 + 1.000 = 3.153.

3) Use the adjusted factor to find the future value of the ordinary annuity of three years: $10,000 × 3.153 = $31,530.

The future value of a $10,000 payment made annually at the **end** of each of three years (an ordinary annuity) is $10,000 × 3.153, or **$31,530**.

Calculating the Future Value of an Annuity Due

To calculate the future value of an annuity due, when payments are made at the beginning of each period, **if an annuity due factor table is available**, use the annuity due factor table as follows:

Future value of an annuity due	**=**	**Periodic payment × FV of an Annuity Due Factor for *n* periods at *i* interest**

However, it is more likely that an annuity due factor table will not be available. To calculate the future value of an annuity due using a Future Value of an Ordinary Annuity table such as the one available at the link on the first page of this document, use the ordinary annuity factor from the table for **one period more** and **subtract 1.000** from it.

Future value of an annuity due	**=**	**Periodic payment × FV of an Ordinary Annuity factor for (*n* + *1* periods at *i* interest) − 1.000**

Example: To find the future value of an annuity due of $10,000 with payments made at the **beginning** of each period for **three** years at an interest rate of 5% when the Future Value of an Annuity table available is for an ordinary annuity (for payments made at the **end** of the periods):

1) Find the ordinary annuity factor for **four** years at 5% in the ordinary annuity table: The ordinary annuity factor for four years at 5% is 4.310.
2) Subtract 1.000 from the factor: 4.310 − 1.000 = 3.310.
3) Use the adjusted factor to find the future value of the annuity due of three years: $10,000 × 3.310 = $33,100.

The future value of a $10,000 payment made annually at the **beginning** of each of three years (an annuity due) is $10,000 × 3.310, or **$33,100**.

The future value of an annuity can also be used to find the amount of the amount of the payment that needs to be invested periodically to have a given amount at the end of a certain period, assuming a certain interest rate. To find how much needs to be invested each period, restate the future value formula to solve for the periodic payment as follows:

Periodic payment	**=**	**Future Value ÷ FV of an Annuity factor for *n* periods at *i* interest**

The factor can be either the factor for an ordinary annuity (if payments are to be made at the end of each period) or the factor for an annuity due (if payments are to be made at the beginning of each period).

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Example: Cole's Strip Mining Company needs to have $5,000,000 in five years to restore the land on which it is currently mining coal. If Cole can earn 6% interest per year on the funds it sets aside from earnings each year, how much does the company need to set aside from earnings at the end of each year to have the needed $5,000,000 at the end of five years?

Note that this is an ordinary annuity because the payments will be made at the **end** of each year. The $5,000,000 amount needed at the end of five years is the future value. The future value, the term, and the interest rate are all known. Solve for the periodic payment.

Periodic payment = Future Value ÷ Future Value of an Ordinary Annuity Factor at 6% for 5 years

The Future Value of an Ordinary Annuity factor for five years at a 6% rate from the table available at the link on the first page of this document is 5.637.

Periodic payment = $5,000,000 ÷ 5.637 = **$886,997**

If Cole decides to make the five annual payments into the fund at the **beginning** of each year instead of at the end of each year, the company will need to use the future value of an annuity due factor to calculate the amount of each annual payment.

Periodic payment = $5,000,000 ÷ Future Value of an Annuity Due Factor at 6% for 5 years

To use the Ordinary Annuity table to find the Annuity Due factor, use the Future Value of an Ordinary Annuity factor for 6% for **six** years, **one period more** than five years, and **subtract 1.000** from it. The ordinary annuity factor for six years is 6.975, and the annuity due factor is 6.975 − 1.000 = 5.975.

Periodic payment = $5,000,000 ÷ (6.975 − 1.000) = **$836,820**

Because the five annual payments are being made at the **beginning** of each period instead of at the end of each period, the amount of each payment can be approximately $50,000 smaller.

Derivation of the Factor for the Future Value of an Ordinary Annuity

If the needed interest rate or term is not available in a factor table, the formula to calculate the future value of an ordinary annuity is:

$$\textbf{Future Value of an Ordinary Annuity} = \frac{(1+i)^n - 1}{i}$$

The resulting factor is the factor for the future value of an **ordinary** annuity.

Example: The calculation of the factor for the future value of a five-year ordinary annuity at an interest rate of 6% per annum compounded annually is

$$\text{Future Value of an Ordinary Annuity at 6\% for 5 years} = \frac{(1+0.06)^5 - 1}{0.06}$$

$$= \frac{1.338225577 - 1}{0.06} = \mathbf{5.63709295}$$

The factor that appears in a Future Value of an Ordinary Annuity table is 5.637.

If the factor for an annuity due (payments made at the beginning of each period) is needed instead and no annuity table is available, calculate the factor for an ordinary annuity of one period more than what is needed using the formula above and subtract 1.000 from it.

Example: Calculation of the factor for the future value of a five-year annuity due by calculating the ordinary annuity factor for one period more than is needed and subtracting 1.000 from it. Using an interest rate of 6% per annum compounded annually for six years, the ordinary annuity factor is

$$\text{Future Value of an Ordinary Annuity at 6\% for 5 years} = \frac{(1 + 0.06)^6 - 1}{0.06}$$

$$= \frac{1.418519111 - 1}{0.06} = \mathbf{6.975318516}$$

The factor for the future value of an ordinary annuity of six years at 6% from the factor table is 6.975.

The ordinary annuity factor for six years can then be adjusted to convert it to a factor for an annuity due for **five** years by subtracting 1 from it: 6.97518516 – 1 = **5.975318516**.